

Symmetries in multi-Higgs model building

Lecture 3: Phenomenological consequences

Igor Ivanov

CFTP, Instituto Superior Técnico, Universidade de Lisboa

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1 A_4 -symmetric 3HDM

2 Fermions in A_4 3HDM

3 A no-go theorem

Working example: A_4

Permutation groups

There is a whole zoo of finite non-abelian groups. The “simplest” examples are:

- S_n , group of all permutations of n elements. Its order is $|S_n| = n!$. The smallest group is $S_2 \simeq \mathbb{Z}_2$. The smallest **non-abelian** is

$$S_3 = \langle a, b | a^2 = b^3 = e, ab = b^2a \rangle.$$

- A_n , group of **even-signature permutations** of n elements; $|A_n| = n!/2$.
- Symmetry groups of regular **polygons** and **polyhedra**:
 - Symmetry group of **equilateral triangle** $\simeq S_3$;
 - Symmetry group of **tetrahedron** $\simeq A_4$;
 - Symmetry group of **cube** $\simeq S_4$.

Irreducible representations of non-abelian groups have $d > 1$.

Group A₄

A₄ is the smallest group with irreducible 3D representation:

$$A_4 = \langle S, T | S^2 = T^3 = e, (ST)^3 = e \rangle, \quad |A_4| = 12.$$

It contains:

- three elements of order 2: S, T^2ST, TST^2 ;
- together with e , they form the Klein subgroup $\mathbb{Z}_2 \times \mathbb{Z}_2$;
- four cycles of order 3 generated by T, ST, TS, T^2ST^2 (8 elements of order 3 in total).

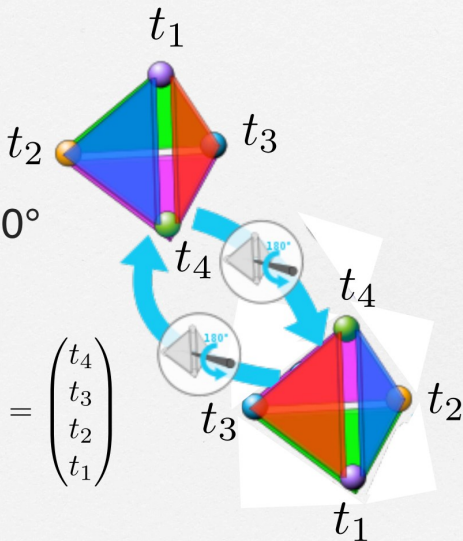
A_4 : transformation S

A_4

• rotation by 180°

S

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = \begin{pmatrix} t_4 \\ t_3 \\ t_2 \\ t_1 \end{pmatrix}$$

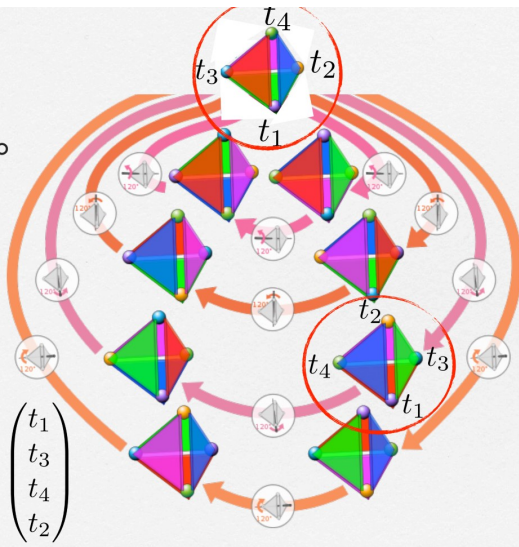


A₄: transformation T A₄

- rotation by 120° anti-clockwise (seen from a vertex)

 T

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_3 \\ t_4 \\ t_2 \end{pmatrix}$$



Group A_4

3D irreducible representation: **diagonal-S basis**

- order 2:

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T^2ST = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad TST^2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- order 3:

$$T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad ST = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix},$$

$$TS = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}, \quad T^2ST^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix},$$

and their squares.

Group A_4

3D irreducible representation: diagonal- T basis

One can switch to another basis in the same 3D space, in which T becomes diagonal.

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \omega \equiv e^{2\pi i/3}, \quad \omega^3 = 1.$$

Then, S takes an “ugly” shape:

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}.$$

Nevertheless, all group multiplications hold: $S^2 = e$, etc

Group A_4

Subspaces in the diagonal- T basis are convenient to define three non-equivalent 1D irreps: $1, 1', 1''$

The full table of all irreps of A_4 :

irrep	S	T
1	$S = 1$	$T = 1$
$1'$	$S = 1$	$T = \omega$
$1''$	$S = 1$	$T = \omega^2$
3	matrix S	matrix T

Notice: only the trivial singlet 1 is invariant under the entire A_4 .

Building symmetry-based models

with the example of A₄-symmetric 3HDM

Tensor product decomposition

Models begin with lagrangian \mathcal{L} , which encodes all interactions.

Terms in the lagrangian are products of various fields:

$$\mathcal{L} = \dots + \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \dots + Y_{ij}^a \overline{Q_{Li}} \Phi_a d_{Rj} + \dots$$

We assume that each set of fields (LH fermions, RH fermions, Higgses, etc) **transforms as a certain representation** of group G .

We want to find which combinations are fully G -invariant.

We must use the **tensor product of representations**.

Tensor product decomposition

Take 3D vectors $a_i = (a_1, a_2, a_3)$ and $b_j = (b_1, b_2, b_3)$ and construct their **tensor product** $a_i b_j$. How does it transform under $SO(3)$ rotations?

$$a_i b_j = \delta_{ij} \frac{(\vec{a}\vec{b})}{3} + \underbrace{\epsilon_{ijk} \cdot v_k}_{=[\vec{a} \times \vec{b}]/2} + \left[\frac{1}{2} (a_i b_j + a_j b_i) - \delta_{ij} \frac{(\vec{a}\vec{b})}{3} \right],$$

which means that inside the 9D tensor $a_i b_j$ there are three invariant subspaces: **singlet**, $\propto \delta_{ij}$; **triplet**, $\propto \epsilon_{ijk} v_k$, and **5-plet**, the traceless symmetric part of $a_i b_j$.

Group-theoretically: $3 \otimes 3 = 1 \oplus 3 \oplus 5$.

This is how tensor product decomposition (= Clebsch-Gordan coeffs) works in the group $SO(3)$.

Tensor product decomposition

For each group, these rules are different (= Clebsch-Gordan coefs are different).

For A_4 , if $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ are two irreducible triplets, then

$$3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_1 \oplus 3_2 .$$

The explicit expressions for their components (in the S -symmetric basis!) are:

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3 ,$$

$$1' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 ,$$

$$1'' = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 ,$$

$$3_1 = (a_2 b_3, a_3 b_1, a_1 b_2) ,$$

$$3_2 = (a_3 b_2, a_1 b_3, a_2 b_1) .$$

The products of **singlets** are intuitive: $1' \otimes 1'' = 1$, etc.

Picking up symmetric terms

When building symmetry-constrained lagrangians, we

- write products of fields, each transforming as a certain irrep of the group G ,
- perform tensor product decomposition,
- out of all final irreps, **keep only trivial singlets** as they are G -symmetric.

In A_4 -symmetric 3HDM, we have three Higgs doublets Φ_1, Φ_2, Φ_3 . In general, the quadratic part of the potential has nine terms $\Phi_i^\dagger \Phi_j$.

But knowing that, for the group A_4 , $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_1 \oplus 3_2$, we keep only the trivial singlet. Therefore, the Higgs potential is

$$V = -m^2 \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3 \right) + V_4$$

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Picking up symmetric terms

For the quartic part, we decompose $(\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l)$,

$$\begin{aligned} [(3 \otimes 3) \otimes (3 \otimes 3)]_{sym} &= [(1 \oplus 1' \oplus 1'' \oplus 3_1 \oplus 3_2) \otimes (1 \oplus 1' \oplus 1'' \oplus 3_1 \oplus 3_2)]_{sym} \\ &= 1 \otimes 1 + 1' \otimes 1'' + \underbrace{(3_1 \otimes 3_1)}_{=1 \oplus \dots} + \underbrace{(3_2 \otimes 3_2)}_{=1 \oplus \dots} + \underbrace{(3_1 \otimes 3_2)}_{=1 \oplus \dots} + \dots, \end{aligned}$$

which gives five trivial singlets **1**:

$$\begin{aligned} V_4 &= \lambda_1 \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3 \right)^2 \\ &+ \lambda_2 \left[(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + (\Phi_2^\dagger \Phi_2)(\Phi_3^\dagger \Phi_3) + (\Phi_3^\dagger \Phi_3)(\Phi_1^\dagger \Phi_1) \right] \\ &+ \lambda_3 \left[(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + (\Phi_2^\dagger \Phi_3)(\Phi_3^\dagger \Phi_2) + (\Phi_3^\dagger \Phi_1)(\Phi_1^\dagger \Phi_3) \right] \\ &+ \left(\lambda_4 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_3)^2 + (\Phi_3^\dagger \Phi_1)^2 \right] + h.c. \right) \end{aligned}$$

Spontaneous symmetry breaking

In this way, we get the **full A₄-symmetric potential in 3HDM**.

But the minimum of this potential (v_1, v_2, v_3) may break this group, fully or completely. Which options are available for the minimum in the A₄-symmetric 3HDM?

It turns out that vevs (v_1, v_2, v_3) **cannot be arbitrary!** Depending on parameters λ , only four **vev alignments** are possible Degee, Ivanov, Keus, 2012:

- $(1, 0, 0)$, residual symmetry $\mathbb{Z}_2 \times \mathbb{Z}_2$.
- $(1, 1, 1)$, residual symmetry \mathbb{Z}_3 .
- $(1, \omega, \omega^2)$, residual symmetry \mathbb{Z}_3 ; remember: we are inside $PSU(3)$!
- $(1, e^{i\alpha}, 0)$, residual symmetry \mathbb{Z}_2 .

Conclusion: **it is impossible** to break the A₄ symmetry completely.

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Majorana neutrino masses in A_4 3HDM

Extending A_4 3HDM to charged leptons

Let us try to extend A_4 symmetry of 3HDM to the **Majorana neutrino mass matrix**, [Gonzales Felipe, Serodio, Silva, 1304.3468](#).

Begin with charged lepton Yukawa interactions:

$$\bar{L}_i Y_{ij}^a \underbrace{\Phi_a}_3 \ell_{Rj} + h.c.$$

We know that $\Phi_a = (\Phi_1, \Phi_2, \Phi_3)$ transforms as triplet **3** under A_4 .

Therefore, the product of L_i and ℓ_{Rj} must also transform as a triplet **3** to produce the trivial singlet **1** at the end.

L_i	ℓ_{Rj}
3	3
(1, 1', 1'')	3
3	(1, 1', 1'')

Extending A₄ 3HDM to charged leptons

For example, if $\bar{L}_i \sim (1, 1', 1'')$ and $l_{Rj} \sim 3$, we get:

$$\begin{aligned}
 \bar{L}_i Y_{ij}^a \Phi_a l_{Rj} &= y_1 \bar{L}_1 \underbrace{\Phi_a l_{Rj}}_1 + y_2 \bar{L}_2 \underbrace{\Phi_a l_{Rj}}_{1''} + y_3 \bar{L}_3 \underbrace{\Phi_a l_{Rj}}_{1'} \\
 &= y_1 \bar{L}_1 (\Phi_1 l_{R1} + \Phi_2 l_{R2} + \Phi_3 l_{R3}) \\
 &\quad + y_2 \bar{L}_2 (\Phi_1 l_{R1} + \omega \Phi_2 l_{R2} + \omega^2 \Phi_3 l_{R3}) \\
 &\quad + y_3 \bar{L}_3 (\Phi_1 l_{R1} + \omega^2 \Phi_2 l_{R2} + \omega \Phi_3 l_{R3})
 \end{aligned}$$

Pick up a vev alignment, for example, $v(1, 1, 1)$. Then, charged lepton mass matrix is

$$M_\ell = v \begin{pmatrix} y_1 & y_1 & y_1 \\ y_2 & \omega y_2 & \omega^2 y_2 \\ y_3 & \omega^2 y_3 & \omega y_3 \end{pmatrix},$$

which, after diagonalization gives $m_\ell = \{y_1 v, y_2 v, y_3 v\} \rightarrow \text{OK}$.

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Extending A_4 3HDM to Majorana neutrinos

Then, include Majorana neutrino terms:

$$c_{ij}^{ab} (L_i^T \tilde{\Phi}_a^*) \mathcal{C} (\tilde{\Phi}_b^\dagger L_j).$$

Group-theoretically, we see

$$(L \otimes L) \otimes (\underbrace{\tilde{\Phi}}_3 \otimes \underbrace{\tilde{\Phi}}_3)$$

Since $\overline{L}_i \sim (1, 1', 1'')$, the product $L \otimes L$ also contains 1, 1', and 1'', which are coupled to $3 \otimes 3$:

$$\begin{aligned} & \frac{g_1}{\Lambda} (L_1 L_1 + L_2 L_3 + L_3 L_2) (\tilde{\Phi}_1 \tilde{\Phi}_1 + \tilde{\Phi}_2 \tilde{\Phi}_2 + \tilde{\Phi}_3 \tilde{\Phi}_3) \\ + & \frac{g_2}{\Lambda} (L_1 L_2 + L_2 L_1 + L_3 L_3) (\tilde{\Phi}_1 \tilde{\Phi}_1 + \omega \tilde{\Phi}_2 \tilde{\Phi}_2 + \omega^2 \tilde{\Phi}_3 \tilde{\Phi}_3) \\ + & \frac{g_3}{\Lambda} (L_1 L_3 + L_2 L_2 + L_3 L_1) (\tilde{\Phi}_1 \tilde{\Phi}_1 + \omega^2 \tilde{\Phi}_2 \tilde{\Phi}_2 + \omega \tilde{\Phi}_3 \tilde{\Phi}_3) \end{aligned}$$

Extending A_4 3HDM to Majorana neutrinos

Next, substituting the chosen vev alignment $(1, 1, 1)$, we get neutrino mass matrix:

$$\mathcal{M}_\nu = \frac{g_1 v^2}{2\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

We obtain **three degenerate neutrinos!**

Conclusion

extending A_4 symmetry to charged leptons and Majorana neutrinos with irrep assignment

$$\Phi \sim 3, \quad L \sim (1, 1', 1''), \quad \ell_R \sim 3$$

and with the vev alignment $\langle \phi^0 \rangle = v(1, 1, 1)$ is **ruled out** by experiment.

Extending A_4 3HDM to Majorana neutrinos

One needs to check all possible irrep assignments and all possible vev alignments. This was done in [Gonzales Felipe, Serodio, Silva, 1304.3468](#).

The result is: **all possible combinations are ruled out experimentally**. The problems can be:

- massless charged leptons,
- degenerate neutrino masses,
- insufficient neutrino mixing.

Thus, 3HDM scalar sector offers too little freedom to produce viable Majorana neutrino masses through the A_4 symmetry group.

One needs to **enlarge the scalar sector** to get a viable neutrino sector → **this is why people introduce flavons**.

Quarks in A₄ 3HDM

Similar problems for quarks, [Gonzales Felipe, Serodio, Silva, 1302.0861](#).

$$-\mathcal{L}_Y = \bar{Q}_L \Gamma_a \phi_a d_R + \bar{Q}_L \Delta_a \tilde{\phi}_a u_R + h.c.$$

which gives after EWSB the quark mass matrices

$$M_d = \Gamma_a v_a / \sqrt{2}, \quad M_u = \Delta_a v_a^* / \sqrt{2}.$$

They can be diagonalized: $V_{dL}^\dagger M_d V_{dR} = D_d$ and $V_{uL}^\dagger M_u V_{uR} = D_u$. It is convenient to define

$$H_d \equiv M_d M_d^\dagger, \quad H_u \equiv M_u M_u^\dagger,$$

which are diagonalized as $V_{dL}^\dagger H_d V_{dL} = D_d^2$ and $V_{uL}^\dagger H_u V_{uL} = D_u^2$. The mismatch $V_{dL} \neq V_{uL}$ produces the **CKM matrix**: $V = V_{uL}^\dagger V_{dL}$.

We want the quark sector to be **minimally viable**:

- quark masses are **non-zero** and **non-degenerate**;
- **non-block-diagonal** CKM matrix V , to allow for three mixing angles,
- **non-zero CP-violating phase** in V .

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Quarks in A_4 3HDM

Consider again A_4 3HDM with $\phi \sim 3$, $Q_L \sim (1, 1', 1'')$, $d_R \sim u_R \sim 3$.
 Then, the Yukawa lagrangian for down sector

$$\begin{aligned} & \alpha_1 \bar{Q}_{L1} (\phi_1 d_{R1} + \phi_2 d_{R2} + \phi_3 d_{R3}) \\ + & \alpha_2 \bar{Q}_{L2} (\phi_1 d_{R1} + \omega \phi_2 d_{R2} + \omega^2 \phi_3 d_{R3}) \\ + & \alpha_3 \bar{Q}_{L2} (\phi_1 d_{R1} + \omega^2 \phi_2 d_{R2} + \omega \phi_3 d_{R3}) \end{aligned}$$

yields the mass matrix

$$M_d = \begin{pmatrix} \alpha_1 v_1 & \alpha_1 v_2 & \alpha_1 v_3 \\ \alpha_2 v_1 & \omega \alpha_2 v_2 & \omega^2 \alpha_2 v_3 \\ \alpha_3 v_1 & \omega^2 \alpha_3 v_2 & \omega \alpha_3 v_3 \end{pmatrix}$$

Similar matrix for the up-quark sector, with parameters $\beta_{1,2,3}$.

Quark masses in A_4 3HDM

Again, let's select vev alignment $v(1, 1, 1)$. Then,

$$M_d = v \begin{pmatrix} \alpha_1 & \alpha_1 & \alpha_1 \\ \alpha_2 & \omega\alpha_2 & \omega^2\alpha_2 \\ \alpha_3 & \omega^2\alpha_3 & \omega\alpha_3 \end{pmatrix}, \quad M_u = v \begin{pmatrix} \beta_1 & \beta_1 & \beta_1 \\ \beta_2 & \omega\beta_2 & \omega^2\beta_2 \\ \beta_3 & \omega^2\beta_3 & \omega\beta_3 \end{pmatrix}$$

Diagonalization of M_d and M_u produces six different quark masses.

So, if this case **fine**?

Quarks in A₄ 3HDM

NO!

Let's calculate H_d and H_u :

$$H_d = 3v^2 \begin{pmatrix} |\alpha_1|^2 & 0 & 0 \\ 0 & |\alpha_2|^2 & 0 \\ 0 & 0 & |\alpha_3|^2 \end{pmatrix}, \quad H_u = 3v^2 \begin{pmatrix} |\beta_1|^2 & 0 & 0 \\ 0 & |\beta_2|^2 & 0 \\ 0 & 0 & |\beta_3|^2 \end{pmatrix}$$

They are already diagonal $\rightarrow V_{dL} = V_{uL} = \delta_{ij} \rightarrow$ **trivial CKM matrix.**

One can check all possible rep assignments and vev alignments \rightarrow the masses/mixing/CPV is **never acceptable**, Gonzales Felipe, Serodio, Silva, 1302.0861.

What is the problem, fundamentally?

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Consider a global symmetry group G which acts non-trivially on quarks and (active) Higgs doublets in arbitrary representations.

A no-go theorem

The only way to obtain a non-block-diagonal CKM mixing matrix and, simultaneously, non-degenerate and non-zero quark masses, is to arrange that $\langle \phi_a \rangle$ break completely the group G .

If **any** residual symmetry from G survives, the quark sector will be unphysical, Gonzalez Felipe, Ivanov, Nishi, Serodio, Silva, 1401.5807.

Historical remark: a less accurate version of this theorem was proved in Leurer, Nir, Seiberg, NPB398 (1993) 319. We found a counterexample to their version and refined the statement.

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The moral

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When building bSM models, do not ignore **unconventional mathematical tools**. They may help you answer questions which traditional “poor physicist’s methods” just cannot handle.