Symmetries in multi-Higgs model building

Lecture 3: Phenomenological consequences

Igor Ivanov

CFTP, Instituto Superior Técnico, Universidade de Lisboa

Korea Institute for Advanced Study, Seoul, April 17, 2018

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Working example: A_4

Permutation groups

There is a whole zoo of finite non-abelian groups. The “simplest” examples are:

- S_n, group of all permutations of n elements. Its order is $|S_n| = n!$. The smallest group is $S_2 \simeq \mathbb{Z}_2$. The smallest non-abelian is

$$S_3 = \langle a, b | a^2 = b^3 = e, ab = b^2 a \rangle \,.$$

- A_n, group of even-signature permutations of n elements; $|A_n| = n!/2$.
- Symmetry groups of regular polygons and polyhedra:
  - Symmetry group of equilateral triangle $\simeq S_3$;
  - Symmetry group of tetrahedron $\simeq A_4$;
  - Symmetry group of cube $\simeq S_4$.

Irreducible representations of non-abelian groups have $d > 1$.

Group A_4

A_4 is the smallest group with irreducible 3D representation:

$$A_4 = \langle S, T | S^2 = T^3 = e, (ST)^3 = e \rangle\,, \quad |A_4| = 12\,.$$

It contains:

- three elements of order 2: S, T^2ST, TST^2;
- together with e, they form the Klein subgroup $\mathbb{Z}_2 \times \mathbb{Z}_2$;
- four cycles of order 3 generated by T, ST, TS, T^2ST^2 (8 elements of order 3 in total).

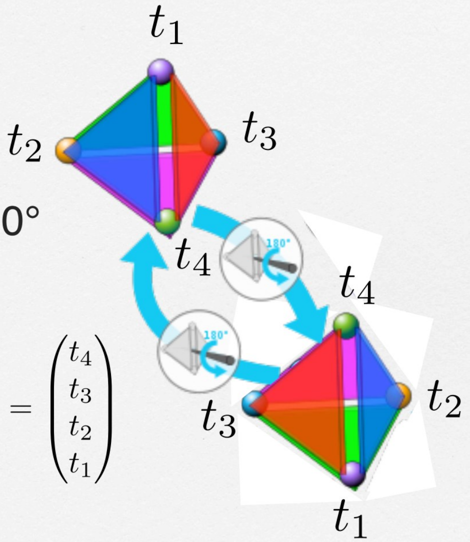
A_4: transformation S

A4

- rotation by 180°

$$S$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = \begin{pmatrix} t_4 \\ t_3 \\ t_2 \\ t_1 \end{pmatrix}$$



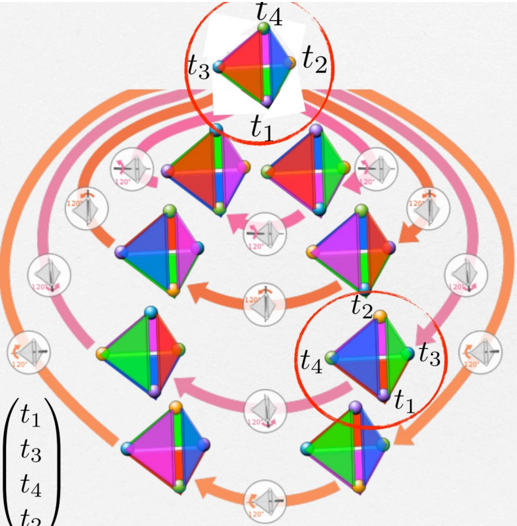
A_4: transformation T

A4

- rotation by 120° anti-clockwise (seen from a vertex)

$$T$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_3 \\ t_4 \\ t_2 \end{pmatrix}$$



Group A_4

3D irreducible representation: diagonal-S basis

- order 2:

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T^2ST = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad TST^2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- order 3:

$$T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad ST = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix},$$

$$TS = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad T^2ST^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix},$$

and their squares.

Group A_4

3D irreducible representation: diagonal-T basis

One can switch to another basis in the same 3D space, in which T becomes diagonal.

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \omega \equiv e^{2\pi i/3}, \quad \omega^3 = 1\,.$$

Then, S takes an "ugly" shape:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}.$$

Nevertheless, all group multiplications hold: $S^2 = e$, etc

Group A_4

Subspaces in the diagonal-T basis are convenient to define three non-equivalent 1D irreps: 1, $1'$, $1''$

The full table of all irreps of A_4:

irrep	S	T
1	$S = 1$	$T = 1$
$1'$	$S = 1$	$T = \omega$
$1''$	$S = 1$	$T = \omega^2$
3	matrix S	matrix T

Notice: only the trivial singlet 1 is invariant under the entire A_4.

Building symmetry-based models

with the example of A_4-symmetric 3HDM

Tensor product decomposition

Models begin with lagrangian $\mathcal{L}$, which encodes all interactions.

Terms in the lagrangian are products of various fields:

$$\mathcal{L} = \cdots + \lambda_5(\Phi_1^\dagger \Phi_2)^2 + \cdots + Y_{ij}^a \overline{Q_{Li}} \Phi_a d_{Rj} + \cdots$$

We assume that each set of fields (LH fermions, RH fermions, Higgses, etc) transforms as a certain representation of group G.

We want to find which combinations are fully G-invariant.

> We must use the tensor product of representations.

Tensor product decomposition

Take 3D vectors $a_i = (a_1, a_2, a_3)$ and $b_j = (b_1, b_2, b_3)$ and construct their tensor product $a_i b_j$. How does it transform under $SO(3)$ rotations?

$$a_i b_j = \delta_{ij} \frac{(\vec{a}\vec{b})}{3} + \epsilon_{ijk} \cdot \underbrace{v_k}_{=[\vec{a}\times\vec{b}]/2} + \left[\frac{1}{2}(a_i b_j + a_j b_i) - \delta_{ij} \frac{(\vec{a}\vec{b})}{3} \right],$$

which means that inside the 9D tensor $a_i b_j$ there are three invariant subspaces: singlet, $\propto \delta_{ij}$; triplet, $\propto \epsilon_{ijk} v_k$, and 5-plet, the traceless symmetric part of $a_i b_j$.

Group-theoretically: $3 \otimes 3 = 1 \oplus 3 \oplus 5$.

This is how tensor product decomposition (= Clebsch-Gordan coefs) works in the group $SO(3)$.

Tensor product decomposition

For each group, these rules are different (= Clebsch-Gordan coefs are different).

For A_4, if $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ are two irreducible triplets, then

$$3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_1 \oplus 3_2 \,.$$

The explicit expressions for their components (in the S-symmetric basis!) are:

$$\begin{aligned}
1 &= a_1 b_1 + a_2 b_2 + a_3 b_3 \,, \\
1' &= a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 \,, \\
1'' &= a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \,, \\
3_1 &= (a_2 b_3, a_3 b_1, a_1 b_2) \,, \\
3_2 &= (a_3 b_2, a_1 b_3, a_2 b_1) \,.
\end{aligned}$$

The products of singlets are intuitive: $1' \otimes 1'' = 1$, etc.

Picking up symmetric terms

When building symmetry-constrained lagrangians, we

- write products of fields, each transforming as a certain irrep of the group G,
- perform tensor product decomposition,
- out of all final irreps, keep only trivial singlets as they are G-symmetric.

In A_4-symmetric 3HDM, we have three Higgs doublets Φ_1, Φ_2, Φ_3. In general, the quadratic part of the potential has nine terms $\Phi_i^\dagger \Phi_j$.

But knowing that, for the group A_4, $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_1 \oplus 3_2$, we keep only the trivial singlet. Therefore, the Higgs potential is

$$V = -m^2 \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3\right) + V_4$$

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Picking up symmetric terms

For the quartic part, we decompose $(\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l)$,

$$[(3\otimes 3)\otimes(3\otimes 3)]_{sym} = [(1\oplus 1'\oplus 1''\oplus 3_1\oplus 3_2)\otimes(1\oplus 1'\oplus 1''\oplus 3_1\oplus 3_2)]_{sym}$$

$$= 1\otimes 1 + 1'\otimes 1'' + \underbrace{(3_1\otimes 3_1)}_{=1\oplus\ldots} + \underbrace{(3_2\otimes 3_2)}_{=1\oplus\ldots} + \underbrace{(3_1\otimes 3_2)}_{=1\oplus\ldots} + \ldots\,,$$

which gives five trivial singlets 1:

$$\begin{aligned} V_4 &= \lambda_1\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3\right)^2 \\ &+ \lambda_2\left[(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_2)(\Phi_3^\dagger\Phi_3) + (\Phi_3^\dagger\Phi_3)(\Phi_1^\dagger\Phi_1)\right] \\ &+ \lambda_3\left[(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + (\Phi_2^\dagger\Phi_3)(\Phi_3^\dagger\Phi_2) + (\Phi_3^\dagger\Phi_1)(\Phi_1^\dagger\Phi_3)\right] \\ &+ \left(\lambda_4\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_3)^2 + (\Phi_3^\dagger\Phi_1)^2\right] + h.c.\right) \end{aligned}$$

Spontaneous symmetry breaking

In this way, we get the full A_4-symmetric potential in 3HDM.

But the minimum of this potential (v_1, v_2, v_3) may break this group, fully or completely. Which options are available for the minimum in the A_4-symmetric 3HDM?

It turns out that vevs (v_1, v_2, v_3) cannot be arbitrary! Depending on paremeters λ, only four vev alignments are possible Degee, Ivanov, Keus, 2012:

- $(1, 0, 0)$, residual symmetry $\mathbb{Z}_2 \times \mathbb{Z}_2$.
- $(1, 1, 1)$, residual symmetry $\mathbb{Z}_3$.
- $(1, \omega, \omega^2)$, residual symmetry $\mathbb{Z}_3$; remember: we are inside $PSU(3)$!
- $(1, e^{i\alpha}, 0)$, residual symmetry $\mathbb{Z}_2$.

Conclusion: it is impossible to break the A_4 symmetry completely.

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Majorana neutrino masses in A_4 3HDM

Extending A_4 3HDM to charged leptons

Let us try to extend A_4 symmetry of 3HDM to the Majorana neutrino mass matrix, Gonzales Felipe, Serodio, Silva, 1304.3468.
Begin with charged lepton Yukawa interactions:

$$\overline{L_i} Y^a_{ij} \underbrace{\Phi_a}_{3} \ell_{Rj} + h.c.$$

We know that $\Phi_a = (\Phi_1, \Phi_2, \Phi_3)$ transforms as triplet 3 under A_4.

Therefore, the product of L_i and ℓ_{Rj} must also transform as a triplet 3 to produce the trivial singlet 1 at the end.

L_i	ℓ_{Rj}
3	3
(1, 1', 1")	3
3	(1, 1', 1")

Extending A_4 3HDM to charged leptons

For example, if $\overline{L_i} \sim (1, 1', 1'')$ and $\ell_{Rj} \sim 3$, we get:

$$\begin{aligned}
\overline{L_i} Y^a_{ij} \Phi_a \ell_{Rj} &= y_1 \overline{L_1} \underbrace{\Phi_a \ell_{Rj}}_{1} + y_2 \overline{L_2} \underbrace{\Phi_a \ell_{Rj}}_{1''} + y_3 \overline{L_3} \underbrace{\Phi_a \ell_{Rj}}_{1'} \\
&= y_1 \overline{L_1} (\Phi_1 \ell_{R1} + \Phi_2 \ell_{R2} + \Phi_3 \ell_{R3}) \\
&\quad + y_2 \overline{L_2} (\Phi_1 \ell_{R1} + \omega \Phi_2 \ell_{R2} + \omega^2 \Phi_3 \ell_{R3}) \\
&\quad + y_3 \overline{L_3} (\Phi_1 \ell_{R1} + \omega^2 \Phi_2 \ell_{R2} + \omega \Phi_3 \ell_{R3})
\end{aligned}$$

Pick up a vev alignment, for example, $v(1, 1, 1)$. Then, charged lepton mass matrix is

$$M_\ell = v \begin{pmatrix} y_1 & y_1 & y_1 \\ y_2 & \omega y_2 & \omega^2 y_2 \\ y_3 & \omega^2 y_3 & \omega y_3 \end{pmatrix},$$

which, after diagonalization gives $m_\ell = \{y_1 v, \, y_2 v, \, y_3 v\} \to$ OK.

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Extending A_4 3HDM to Majorana neutrinos

Then, include Majorana neutrino terms:

$$c_{ij}^{ab}(L_i^T\tilde{\Phi}_a^*)\,\mathcal{C}\,(\tilde{\Phi}_b^\dagger L_j)\,.$$

Group-theoretically, we see

$$(L\otimes L)\otimes(\underbrace{\tilde{\Phi}}_{3}\otimes\underbrace{\tilde{\Phi}}_{3})$$

Since $\overline{L_i}\sim(1,1',1'')$, the product $L\otimes L$ also contains 1, 1′, and 1″, which are coupled to $3\otimes 3$:

$$\begin{aligned}
&\frac{g_1}{\Lambda}(L_1L_1+L_2L_3+L_3L_2)(\tilde{\Phi}_1\tilde{\Phi}_1+\tilde{\Phi}_2\tilde{\Phi}_2+\tilde{\Phi}_3\tilde{\Phi}_3)\\
+&\frac{g_2}{\Lambda}(L_1L_2+L_2L_1+L_3L_3)(\tilde{\Phi}_1\tilde{\Phi}_1+\omega\tilde{\Phi}_2\tilde{\Phi}_2+\omega^2\tilde{\Phi}_3\tilde{\Phi}_3)\\
+&\frac{g_3}{\Lambda}(L_1L_3+L_2L_2+L_3L_1)(\tilde{\Phi}_1\tilde{\Phi}_1+\omega^2\tilde{\Phi}_2\tilde{\Phi}_2+\omega\tilde{\Phi}_3\tilde{\Phi}_3)
\end{aligned}$$

Extending A_4 3HDM to Majorana neutrinos

Next, substituting the chosen vev alignment $(1,1,1)$, we get neutrino mass matrix:

$$\mathcal{M}_\nu = \frac{g_1 v^2}{2\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .$$

We obtain three degenerate neutrinos!

Conclusion

extending A_4 symmetry to charged leptons and Majorana neutrinos with irrep assignment

$$\Phi \sim 3\,, \quad L \sim (1, 1', 1'')\,, \quad \ell_R \sim 3$$

and with the vev alignment $\langle \phi^0 \rangle = v(1,1,1)$ is ruled out by experiment.

Extending A_4 3HDM to Majorana neutrinos

One needs to check all possible irrep assignments and all possible vev alignments. This was done in Gonzales Felipe, Serodio, Silva, 1304.3468.

The result is: all possible combinations are ruled out experimentally. The problems can be:

- massless charged leptons,
- degenerate neutrino masses,
- insufficient neutrino mixing.

Thus, 3HDM scalar sector offers too little freedom to produce viable Majorana neutrino masses through the A_4 symmetry group.

One needs to enlarge the scalar sector to get a viable neutrino sector $\rightarrow$ this is why people introduce flavons.

Quark sector in A_4 3HDM

Quarks in A_4 3HDM

Similar problems for quarks, Gonzales Felipe, Serodio, Silva, 1302.0861.

$$-\mathcal{L}_Y = \bar{Q}_L \Gamma_a \phi_a d_R + \bar{Q}_L \Delta_a \tilde{\phi}_a u_R + h.c.$$

which gives after EWSB the quark mass matrices

$$M_d = \Gamma_a v_a/\sqrt{2}\,, \quad M_u = \Delta_a v_a^*/\sqrt{2}\,.$$

They can be diagonalized: $V_{dL}^\dagger\, M_d\, V_{dR} = D_d$ and $V_{uL}^\dagger\, M_u\, V_{uR} = D_u$. It is convenient to define

$$H_d \equiv M_d M_d^\dagger\,, \quad H_u \equiv M_u M_u^\dagger\,,$$

which are diagonalized as $V_{dL}^\dagger\, H_d\, V_{dL} = D_d^2$ and $V_{uL}^\dagger\, H_u\, V_{uL} = D_u^2$. The mismatch $V_{dL} \neq V_{uL}$ produces the CKM matrix: $V = V_{uL}^\dagger V_{dL}$.

We want the quark sector to be minimally viable:

- quark masses are non-zero and non-degenerate;
- non-block-diagonal CKM matrix V, to allow for three mixing angles,
- non-zero CP-violating phase in V.

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Quarks in A_4 3HDM

Consider again A_4 3HDM with $\phi \sim 3,\ Q_L \sim (1,\ 1',\ 1''),\ d_R \sim u_R \sim 3$.
Then, the Yukawa lagrangian for down sector

$$\begin{aligned} & \alpha_1 \overline{Q}_{L1}(\phi_1 d_{R1} + \phi_2 d_{R2} + \phi_3 d_{R3}) \\ + \quad & \alpha_2 \overline{Q}_{L2}(\phi_1 d_{R1} + \omega\phi_2 d_{R2} + \omega^2\phi_3 d_{R3}) \\ + \quad & \alpha_3 \overline{Q}_{L2}(\phi_1 d_{R1} + \omega^2\phi_2 d_{R2} + \omega\phi_3 d_{R3}) \end{aligned}$$

yields the mass matrix

$$M_d = \begin{pmatrix} \alpha_1 v_1 & \alpha_1 v_2 & \alpha_1 v_3 \\ \alpha_2 v_1 & \omega\alpha_2 v_2 & \omega^2\alpha_2 v_3 \\ \alpha_3 v_1 & \omega^2\alpha_3 v_2 & \omega\alpha_3 v_3 \end{pmatrix}$$

Similar matrix for the up-quark sector, with parameters $\beta_{1,2,3}$.

Quark masses in A_4 3HDM

Again, let's select vev alignment $v(1,1,1)$. Then,

$$M_d = v \begin{pmatrix} \alpha_1 & \alpha_1 & \alpha_1 \\ \alpha_2 & \omega\alpha_2 & \omega^2\alpha_2 \\ \alpha_3 & \omega^2\alpha_3 & \omega\alpha_3 \end{pmatrix}, \quad M_u = v \begin{pmatrix} \beta_1 & \beta_1 & \beta_1 \\ \beta_2 & \omega\beta_2 & \omega^2\beta_2 \\ \beta_3 & \omega^2\beta_3 & \omega\beta_3 \end{pmatrix}$$

Diagonalization of M_d and M_u produces six different quark masses.

So, if this case fine?

Quarks in A_4 3HDM

NO!

Let's calculate H_d and H_u:

$$H_d = 3v^2 \begin{pmatrix} |\alpha_1|^2 & 0 & 0 \\ 0 & |\alpha_2|^2 & 0 \\ 0 & 0 & |\alpha_3|^2 \end{pmatrix}, \quad H_u = 3v^2 \begin{pmatrix} |\beta_1|^2 & 0 & 0 \\ 0 & |\beta_2|^2 & 0 \\ 0 & 0 & |\beta_3|^2 \end{pmatrix}$$

They are already diagonal $\to V_{dL} = V_{uL} = \delta_{ij} \to$ trivial CKM matrix.

One can check all possible rep assignments and vev alignments $\to$ the masses/mixing/CPV is never acceptable, Gonzales Felipe, Serodio, Silva, 1302.0861.

What is the problem, fundamentally?

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A no-go theorem

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Consider a global symmetry group G which acts non-trivially on quarks and (active) Higgs doublets in arbitrary representations.

A no-go theorem

The only way to obtain a non-block-diagonal CKM mixing matrix and, simultaneously, non-degenerate and non-zero quark masses, is to arrange that $\langle \phi_a \rangle$ break completely the group G.

If any residual symmetry from G survives, the quark sector will be unphysical, Gonzalez Felipe, Ivanov, Nishi, Serodio, Silva, 1401.5807.

Historical remark: a less accurate version of this theorem was proved in Leurer, Nir, Seiberg, NPB398 (1993) 319. We found a counterexample to their version and refined the statement.

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The moral

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When building bSM models, do not ignore unconventional mathematical tools. They may help you answer questions which traditional "poor physicist's methods" just cannot handle.