

# Lecture Series on Topics in PDEs

## Dates:

(First week) 2017.02.13~2017.02.17

(Second week) 2017.02.20~2017.02.24

## Venues:

(First week) Seminar Room 8101, KIAS

(Second week) Seminar Room 8309, KIAS

## Invited Speakers

Woocheol Choi (KIAS)

Younghun Hong (Yonsei University)

Hwakil Kim (KIAS)

Seonghak Kim (Renmin University of China)

Seunghyeok Kim (KIAS)

Youngae Lee (NIMS)

Sungjin Oh (KIAS)

Jihoon Ok (KIAS)

Minsuk Yang (KIAS)

Seokbae Yun (Sungkyunkwan University)

# Time table

	13 (Mon)	14 (Tue)	15 (Wed)	16 (Thu)	17 (Fri)
9:30~11:00	Hwakil Kim				
11:00~12:30	Younghun Hong	Younghun Hong	Younghun Hong	Jihoon Ok	Jihoon Ok
13:30~15:00	Seonghak Kim	Seonghak Kim	Seonghak Kim	Youngae Lee	Youngae Lee
15:00~16:30	Seokbae Yun	Seokbae Yun	Seokbae Yun	Woocheol Choi	Woocheol Choi

	20 (Mon)	21 (Tue)	22 (Wed)	23 (Thu)	24 (Fri)
9:30~11:00	Sungjin Oh	Sungjin Oh			
11:00~12:30	Jihoon Ok	Jihoon Ok	Sungjin Oh	Sungjin Oh	Sungjin Oh
13:30~15:00	Woocheol Choi	Seunghyeok Kim	Minsuk Yang	Minsuk Yang	Minsuk Yang
15:00~16:30	Seunghyeok Kim	Woocheol Choi	Woocheol Choi	Seunghyeok Kim	Seunghyeok Kim

# MATHEMATICAL FOUNDATION FOR THE MAXWELL EQUATION ON BOUNDED DOMAINS

WOOCHEOL CHOI,  
SCHOOL OF MATHEMATICS,  
KOREA INSTITUTE FOR ADVANCED STUDY

In this lecture, I will explain the mathematical background of the Maxwell equation posed on bounded domains. The first part is focused on the time independent case. We shall study the existence theory using the Helmholtz decomposition and the Fredholm alternative.

The second part is devoted to study the well-posedness for the time dependent case. In the last part, we shall review various model equations related to the Maxwell equations.

Key words:  $\text{Curl}(\Omega)$ ,  $\text{Div}(\Omega)$  spaces, Fredholm Alternative, Helmholtz decomposition.

**GLOBAL WELL-POSEDNESS FOR THE NONLINEAR  
HARTREE EQUATION FOR INFINITELY MANY  
PARTICLES**

YOUNGHUN HONG,  
DEPARTMENT OF MATHEMATICS,  
YONSEI UNIVERSITY

The nonlinear Hartree equation for  $N$  particles is given by a system of equations for  $N$  orthonormal functions. In the Heisenberg picture, this equation is described as a single operator-valued PDE, and it has a much richer structure than the system. For instance, the operator-valued equation has stationary solutions, which are simply Fourier multiplier, having infinite many particle number. They include some physically important examples describing the thermal equilibrium. In this talk, we discuss the local and global well-posedness problem on perturbations from these stationary solutions. For the suitable local theory, we employ some Strichartz estimates for operators, sometimes rephrased as Strichartz estimates for orthonormal functions. For global well-posedness, we use the notion of the relative (free) energy and the relative entropy. This problem has been first introduced by Lewin and Sabin recently in 2013, and then improved in several aspects in the joint work with Thomas Chen and Natasa Pavlovic at University of Texas at Austin.

# INTRODUCTION TO OPTIMAL MASS TRANSPORTATION PROBLEM

HWAKIL KIM,  
SCHOOL OF MATHEMATICS,  
KOREA INSTITUTE FOR ADVANCED STUDY

In this lecture, we introduce the mass transportation problem and show how we solve it. It turns out that the mass transportation problem induces the Wasserstein space which provides a nice theoretical framework for studying PDEs describing evolution of a conserved quantity. During the lecture, we will go through the paper “The variational formulation of the Fokker-Planck equation” by Jordan-Kinderlehrer-Otto which opens the connection between mass transportation theory and the evolution type of PDEs.

# GENERALIZED CONVEXITIES IN THE CALCULUS OF VARIATIONS AND SOME APPLICATIONS TO PDES

SEONGHAK KIM,  
RENMIN UNIVERSITY OF CHINA

In the first lecture, I will introduce several generalized convexities related to the minimization problem in the calculus of variations and prove some relations and properties among such convexities. As a main result, the equivalence of weak (or weak\*) lower semi-continuity of the functional and quasi-convexity of the integrand will be explained without proof. In the second lecture, I will recast a general Hamilton-Jacobi PDE as a partial differential inclusion and show how a certain in-approximation scheme can be utilized to extract a.e.-solutions to the PDE in the framework of convex integration and Baire's category methods. We will also see that rank-one convexity plays a special role in solving such a PDE. In the last lecture, I will briefly introduce some results in parabolic PDEs of mixed type, in non-convex elastodynamics and in scalar conservation laws, and choose one of them to explain in detail how the method of convex integration can be used to deduce existence and properties of solutions to some problem.

# CONSTRUCTION OF THE NONLOCAL DELAUNAY HYPERSURFACES

SEUNGHYEOK KIM,  
SCHOOL OF MATHEMATICS,  
KOREA INSTITUTE FOR ADVANCED STUDY

Firstly, we define the nonlocal mean curvature (NMC) for hypersurfaces in the Euclidean space and observe some basic properties. Then we study the result of X. Cabre, M. M. Fall and T. Weth (arXiv:1602.02623) on the existence of the nonlocal Delaunay-type cylinders which have constant NMCs. Its proof is based on the Crandall-Rabinowitz bifurcation theorem.

# UNIFORM ESTIMATES AND BLOW UP BEHAVIOR FOR MEAN FIELD EQUATIONS

YOUNGAE LEE,  
NATIONAL INSTITUTE FOR MATHEMATICAL SCIENCES

In this talk, we study basic properties for mean field type equations. First of all, we review Brezis-Merle triloggy theorem on bounded domain. To understand blow up solutions, we also discuss about the classification of the entire solutions.

# ON THE ENERGY CRITICAL YANG-MILLS EQUATION

SUNG-JIN OH,  
SCHOOL OF MATHEMATICS,  
KOREA INSTITUTE FOR ADVANCED STUDY

In this lecture series, I will describe some recent works concerning the Yang-Mills equation in the energy-critical dimension (i.e., on the  $(4+1)$ -dimensional Minkowski spacetime) with a particular emphasis on the so-called threshold conjecture. Attacking this conjecture requires a wide array of tools such as harmonic analysis, parametrix construction by conjugation with a pseudo-differential operator, geometric (parabolic) flows, monotonicity formulae and blow-up techniques. After a general introduction to the subject, I will attempt to give an introduction of each of these aspects.

# REGULARITY THEORY OF THE $p$ -LAPLACE EQUATIONS

JIHOON OK,  
SCHOOL OF MATHEMATICS,  
KOREA INSTITUTE FOR ADVANCED STUDY

We will discuss on the regularity for weak solutions to the  $p$ -Laplace equations and quasi-minimizers to functionals with  $p$ -growth. Specially, I would like to introduce the  $C^\alpha$ -regularity, the Lipschitz regularity and, if possible, the Calderón-Zygmund estimates

# LOCAL REGULARITY FOR THE NAVIER–STOKES EQUATIONS

MINSUK YANG,  
SCHOOL OF MATHEMATICS,  
KOREA INSTITUTE FOR ADVANCED STUDY

We will review very basic local regularity theory for the Navier-Stokes equations. Our aim is to cover standard (classical) regularity results on linear stationary (non-stationary) Stokes system and nonlinear stationary (non-stationary) Navier-Stokes equations.

# INTRODUCTION TO KINETIC EQUATIONS

SEOKBAE YUN,  
DEPARTMENT OF MATHEMATICS,  
SUNGKYUNKWAN UNIVERSITY

Kinetic equations are mathematical rules that describes the evolution of statistical distribution of interacting particle systems.

In this lecture, introductory materials on kinetic equations are reviewed. We first consider the derivation, and basic mathematical properties of two most famous kinetic equations: The Boltzmann equation and the Vlasov equation. We then study the existence and asymptotic behavior of a simple kinetic model equation.

Lectures 1, 2:

Boltzmann equation and Vlasov equation: derivation and basic properties.

Lecture 3:

Existence and Asymptotic theory for a simple kinetic model.